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Essays In Unemployment And Unemployment Insurance

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ABSTRACT

This thesis examines several issues surrounding the causes of aggregate unemployment that relate to both individual work, rest, and search activities, and inter-industry labor reallocation process. On the other hand, this thesis also explores how heterogenous agents (employed and unemployed) react to a given unemployment insurance program as well as its welfare consequences within a general equilibrium search model.

The research in the first chapter is motivated by the three key features of the employment process in the U.S. economy: (1) job creation is procyclical, (2) job destruction is countercyclical, and (3) job creation is less volatile than job destruction. These features are also found at the sectoral (goods vs services) level. The analysis seeks to explain movements in labor market aggregates as the outcome of the interaction of aggregate and sectoral shocks. This analysis is developed in a multi-sector dynamic competitive general equilibrium model with the following features: (i) each market sector is subject to both aggregate and sectoral disturbances, (ii) it takes time to reallocate labor across sectors, and (iii) labor is indivisible.

The model reproduces the cyclical pattern of job creation and reallocation displayed in the U.S. data relatively well. Workers flow between sectors as jobs are created and destroyed in response to both aggregate and sectoral- specific disturbances. The main finding is that sectoral disturbances are a quantitatively important determinant of aggregate unemployment.

The second chapter develops an economy in which agents differ in their levels of wealth and face different employment opportunities. Individual agents choose to work and search to maximize their lifetime expected utility. The aim here is to evaluate quantitatively a set of feasible fiscal policies and to discuss their welfare consequences both for the employed and the unemployed. The main finding is that UI can improve

social welfare but the distortion created by the corresponding taxes might also cause the decline in social welfare.

Two contributions are made in this chapter. First, individual search activity is modeled in a very different way from that observed in the existing search literature. This model concentrates on the utility-maximization job search and the employment-unemployment experience of risk-averse agents within the context of a dynamic general equilibrium model. Second, when individual agents can only insure themselves by precautionary savings, taxes will directly discourage job search. These taxes also discourage job serch indirectly by distorting the allocation of physical capital and lowering the expected return to job search.

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Chapter 1

The Cyclical Behavior of Job Creation and Job Destruction: A Sectoral Model

1.1 Introduction

What determines the amount of employment in an economy and its distribution across sectors, or the size of the labor market and its breakdown between those with and without jobs. In U.S. economy job creation is procyclical, job destruction is countercyclical, and job creation is less volatile than job destruction. In a well-known paper, Lilien (1982) advanced the hypothesis that variations in sectoral opportunities together with frictions impeding the inter-sector movement of workers play an important role in determining labor market aggregates, and in particular unemployment. The questions raised by these findings are: Can a multisector dynamic general equilibrium model replicate the pattern of job creation, and destruction that is observed in the U.S. data? Are sectoral shocks important for determining the average rate of unemployment?

The analysis seeks to explain movements in labor market aggregates as the outcome of the interaction of aggregate and sectoral shocks. The model developed to do this is a multi-sector dynamic competitive general equilibrium framework. The model has three key features. First, each market sector gets hit by both aggregate and sectoral shocks. This is similar to the classic Long and Plosser (1983) real business cycle model. Second, it takes time to reallocate labor across sectors. Each sector in the market economy can draw new employees from a pool of unemployed workers seeking a job. This pool is made up of agents who entered it in some earlier period, either because they lost their job in a market sector or left the home sector. This feature of the analysis requiring a time cost for job reallocation bears some resemblance to the well-known Lucas and Prescott (1974) equilibrium search model. Third, following Hansen (1985) and Rogerson (1988), it is assumed that labor is indivisible. This assumption ensures that the options of working, searching and staying at home are mutually exclusive.

The model developed reproduces the cyclical pattern of job creation, destruction and reallocation displayed in the U.S. data relatively well. Workers flow between sectors as jobs are created and destroyed in response to both aggregate and sectoral-specific shocks. A conclusion of the paper is that aggregate and sectoral shocks contribute a non-negligible amount to the average level of unemployment.[1] Here, approximately one percentage point of the unemployment rate can be accounted for by aggregate and sectoral shocks. The fact that generally some workers are unemployed, but ready to work, allows sectors to expand their output more rapidly in reaction to favorable circumstances in much the same way as inventories of raw materials, parts, etc. do.[2]

The rest of the paper sets out the model in detail and explores its features quantitatively.

1.2 Model

The multisector dynamic general equilibrium model to be simulated will now be developed.

1.2.1 Economic Environment

A continuum of ex ante identical agents is distributed uniformly over the unit interval. In period t an agent can work in one of two productive sectors, search for a job, or stay at home. To describe this, let $\pi_{i,t}$ represent the fraction of agents who are

[1] Andolfatto [1] studies the equilibrium determination of unemployment within the context of a matching model (that has both aggregate and idiosyncratic shocks).

[2] Clearly technological advances, such as changes in organizational forms, that allow inputs to be allocated more quickly to their end-uses are likely to be desirable.

working in sector i at t, and $\pi_{3,t}$ denote the fraction of agents who are searching. Thus, the fraction of the population currently at home is $1-\sum_{i=1}^{3}\pi_{i,t}$ while $1-\sum_{i=1}^{2}\pi_{i,t}$ is the proportion not working. A description of tastes, technology and the stochastic structure of the model follows.

Tastes

Let c_{it} represent an agent's period-t consumption of the commodity produced in sector i. An agent has one unit of non-sleeping time. Labor effort is indivisible with it being assumed that work and search require w and s hours of effort, respectively. Leisure is then given by $1-l_t$, where $l_t \in \{0, s, w\}$. An agent's expected lifetime utility is given by

$$E\left[\sum_{t=0}^{\infty}\beta^t\{A\ln[\sum_{i=1}^{2}\theta_i c_{it}^{\rho}(l_t)]^{\frac{1}{\rho}}+(1-A)\ln(1-l_t)\}\right], \tag{1.1}$$

where $\beta\in(0,1)$, $\rho\in(-\infty,0)\cup(0,1]$, $\theta_i\in(0,1)$, and $\sum_{i=1}^{2}\theta_i=1$.[3]

Production Technology

Sector i is subject to both aggregate, z_t, and sectoral, $\varepsilon_{i,t}$, disturbances. There is a firm in each sector i that produces output y_{it} according to the production technology:

$$y_{i,t}=z_t\varepsilon_{i,t}(h_{i,t})^{\alpha_i}-I_i(z_t,\varepsilon_{i,t}), \tag{1.2}$$

[3]The case where $\rho=0$ is easily handled by letting the expected value of lifetime ulitity read $E\left[\sum_{t=0}^{\infty}\beta^t\{A\ln[\prod_{i=1}^{2}c_{it}(l_t)^{\theta_i}]+(1-A)\ln(1-l_t)\}\right]$.

where

$$h_{i,t} = w[\pi_{i,t} - \gamma_i(\max[\pi_{i,t} - \pi_{i,t-1}, 0])^{\lambda_i}]. \tag{1.3}$$

In (1.2) $h_{i,t}$ represents the amount of labor input hired by the firm. Hiring new labor is costly. One of the costs is assimilating new workers into the production process, a feature portrayed by (1.3). This cost is increasing in the number of workers that join the firm. This is equivalent to saying that when new workers are hired in a period they are less productive than experienced workers. The term $I_i(z_t, \varepsilon_{i,t})$ is an output-reducing shock. This function is discussed in more detail below. By substituting (1.3) into (1.2) it is easy to see that production is governed by

$$y_{i,t} = z_t \varepsilon_{i,t} w^{\alpha_i}[\pi_{i,t} - \gamma_i(\max[\pi_{i,t} - \pi_{i,t-1}, 0])^{\lambda_i}]^{\alpha_i} - I_i(z_t, \varepsilon_{i,t}). \tag{1.4}$$

Firms are owned by households.

Search Technology

In order to increase its employment a firm must draw new labor from the search pool. Thus, the increase in employment that can occur in a sector is limited by the size of the existing search pool. Specifically,

$$\sum_{i=1}^{2} \max\{0, \pi_{t+1,i} - \pi_{t,i}\} \leq \pi_{t,3}. \tag{1.5}$$

Note that (1.5) implies any reallocation of agents between sectors 1 and 2 will involve a one period transition cost.

Stochastic Structure

The aggregate and sectoral disturbances are independent of one another and follow finite-state first-order Markov processes with supports $Z = \{z^1, z^2, ..., z^m\}$ and $E_i = \{\varepsilon_i^1, ..., \varepsilon_i^m\}$, respectively. Furthermore, it will be assumed that the shocks in sector 1 and 2 are inversely related to one another. In particular, let $\varepsilon_1 = 1/\varepsilon_2 \equiv \varepsilon$.

1.2.2 Planner's Problem

Following Rogerson (1988) and Hansen (1985) the representative household's choice set is extended to include the possibility of a lottery over their consumption and labor allocations. One can think about the lottery mechanism as an employment contract specifying for each $l \in \{0, s, w\}$ a (state-contingent) probability $\pi(l)$ that the agent will work x hours, consume $c_i(l)$ units of sector-i output and enjoy $1 - l$ units of leisure. Since all agents are alike initially, it follows from using an appropriate law of large numbers that $\pi(0) = 1 - \sum_{i=1}^{3} \pi_i$, $\pi(s) = \pi_3$, and $\pi(w) = \sum_{i=1}^{2} \pi_i$. The planner's dynamic programming problem that determines the form of this contract is shown below.

$$V(\pi;z,\varepsilon) = \max_{\{c_i(l)\},\{\pi_i'\}} \left\{ \left(1-\sum_{i=1}^{3}\pi_i'\right)\left[\frac{A}{\rho}\ln\left(\sum_{i=1}^{2}\theta_i\cdot c_i^{\rho}(0)\right)\right] \right.$$

$$+\pi_3'\left[\frac{A}{\rho}\ln\left(\sum_{i=1}^{2}\theta_i\cdot c_i^{\rho}(s)\right)+(1-A)\cdot\ln(1-s)\right] \qquad P(1)$$

$$+\left(\sum_{i=1}^{2}\pi_i'\right)\left[\frac{A}{\rho}\ln\left(\sum_{i=1}^{2}\theta_i\cdot c_i^{\rho}(w)\right)+(1-A)\cdot\ln(1-w)\right]$$

$$\left. +\beta E\left[V(\pi';z',\varepsilon')\mid\pi;z,\varepsilon\right]\right\}$$

subject to

$$(1-\sum_{j=1}^{3}\pi_j')c_i(0)+(\sum_{j=1}^{2}\pi_j')c_i(w)+\pi_3'c_i(s) \qquad (1.6)$$

$$= z\varepsilon_i w^{\alpha_i}\left[\pi_i'-\gamma_i\left(\max\left[\pi_i'-\pi_i,0\right]\right)^{\lambda_i}\right]^{\alpha_i}-I_i(z,\varepsilon_i),\ \text{for } i=1,2,$$

$$\sum_{i=1}^{3}\pi_i' \leq 1, \qquad (1.7)$$

$$\sum_{i=1}^{2}\max\{0,\pi_i'-\pi_i\} \leq \pi_3, \qquad (1.8)$$

$$\pi_i' \geq 0,\ \text{for } i=1,2,3. \qquad (1.9)$$

The resource constraint for each sector is given by (1.6). The next constraint limits the aggregate amount of labor that can be used in non-leisure activities. Equation (1.8) states that the amount of new labor that can be hired by sectors 1 and 2 is restricted by the size of the search pool.

Given the separability of preferences, the planner will select consumption paths that are independent of agents' labor market status.[4] Thus, P(1) can be simplified to

$$
\begin{aligned}
V(\pi; z, \varepsilon) \;=\; & \max_{\{\pi_i'\}} \left\{ \frac{A}{\rho} \ln \left[\sum_{i=1}^{2} \theta_i (z \varepsilon_i w^{\alpha_i} [\pi_i' - \gamma_i (\max[\pi_i' - \pi_i, 0])^{\lambda_i}]^{\alpha_i} - I_i(z, \varepsilon_i))^{\rho} \right] \right. \\
& + (1 - A) \left[\pi_3' \ln(1 - s) + \left(\sum_{i=1}^{2} \pi_i' \right) \ln(1 - w) \right] \\
& \left. + \beta \cdot E \left[V(\pi'; z', \varepsilon') \mid \pi; z, \varepsilon \right] \right\}.
\end{aligned} \qquad P(2)
$$

subject to (1.7), (1.8), and (1.9).

1.2.3 Discussion

Multisector frameworks similar to the one presented above have been developed in Rogerson (1987) and Hornstein (1991). The planning problem P(1) determines a Pareto-optimal allocation for the economy under study. An interesting question that arises is whether or not this Pareto-optimal allocation can be decentralized as a competitive equilibrium? By extending the analysis of Prescott and Rios-Rull (1992), it should be possible to show that this allocation can be supported as a quasi-competitive equilibrium. A key step in doing this is to represent the commodity space as a set of infinite sequences of measures specifying the odds of consuming a given quantity of goods and leisure, contingent upon a particular history of aggregate and sectoral shocks.

[4]For more detail, see Greenwood and Huffman (1988) or Rogerson and Wright (1988).

1.3 Calibration

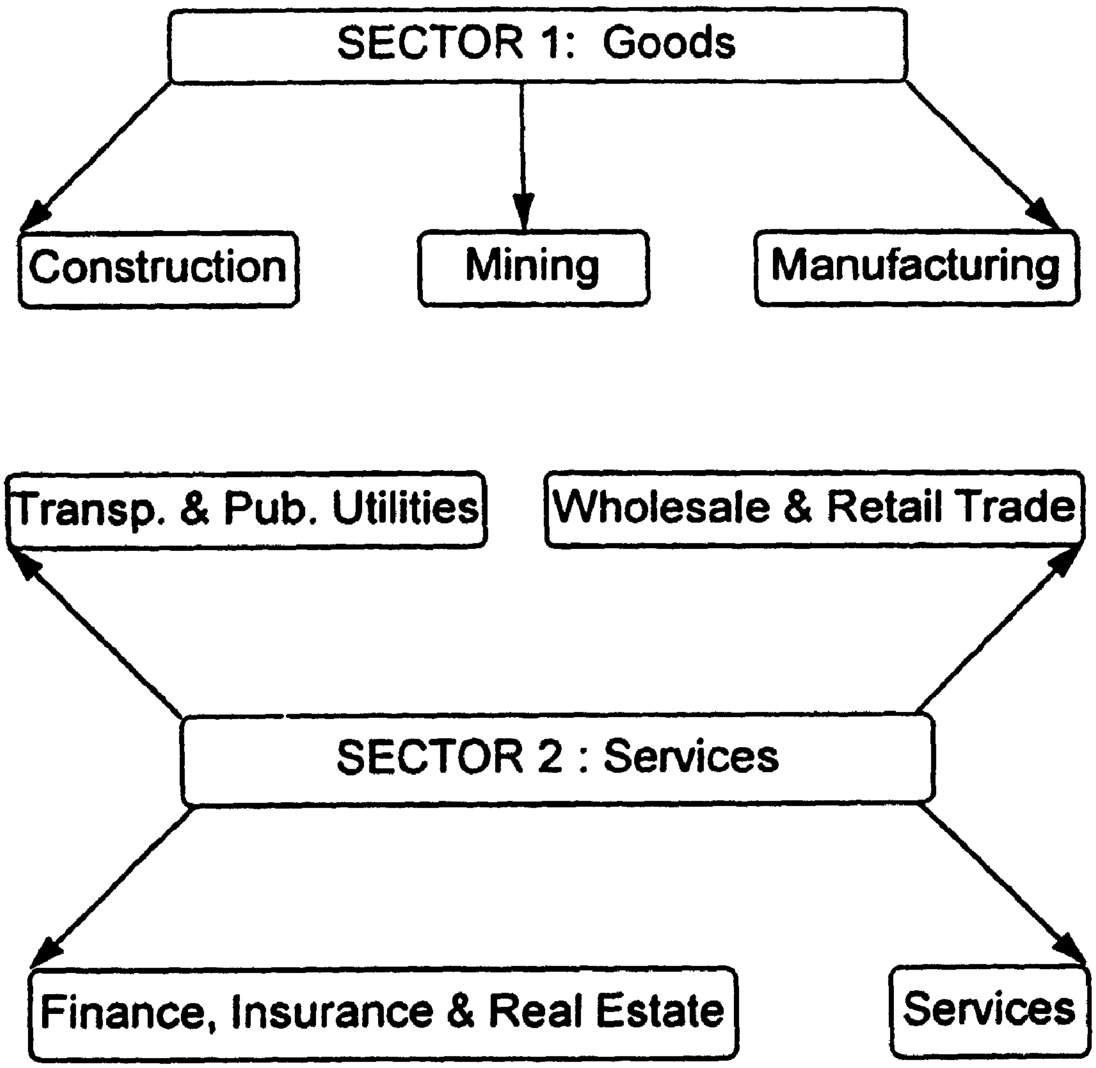
The model is restricted to two sectors, assumed to correspond to the goods and service sectors of the U.S. economy. The industries that make up these sectors are shown in Figure 1; one period is assumed to be one quarter.

1.3.1 Preference Parameters

The quarterly interest rate is taken to be one percent; thus the discount factor, β, is 0.99. Next, data from the *Monthly Labor Review* shows that, on average, the employed work 39 out of the approximately 100 non sleeping hours per week available to them; consequently, $w = .39$. According to Barron and Mellow (1979), the mean number of hours spent searching per week is approximately 7 which implies $s = .07$. In a similar vein, a value of 0.28 was picked for the coefficient A in the utility function. This results in approximately 25% of aggregate non sleeping hours being spent at work. In the U.S. data the goods sector is about 58 percent of the size of the service sector, when measured by employment. This occurs in the model's steady state if $\theta_1 = .43$ ($\theta_2 = .57$). Finally, the parameter $\rho \in (-\infty, 1]$ governs the amount of substitution between goods and services in the utility function. Independent evidence on an appropriate value for ρ is hard to come by. In the subsequent analysis, ρ is assigned a value of 0.55.[5]

[5] The utility function specified in (1.1) implies that an agent will divide his consumption between goods and services according to the formula $\ln \frac{c_1}{c_2} = \frac{1}{\rho - 1} \ln p$, where p is the relative price of goods in terms of services. Estimation of this equation using instrumental variables yielded a value of .55 for ρ. Unfortunately, this point estimate was insignificant at the 95% level of confidence.

Figure 1.1. Definition of Sectors



1.3.2 Technology Parameters

The two production function parameters, α_1 and α_2, are set equal to 0.74 and 0.64 respectively. These numbers are labor's share of income in goods and services sectors for the 1964-1987 period.[6]

1.3.3 Adjustment Costs

The adjustment cost parameters, λ_i and γ_i, are set at 2.0 and 5.5, respectively, for both sectors. These are free parameters that determine the speed of sectoral employment adjustment.[7]

1.3.4 Shocks

Recall the assumption that $\varepsilon_1 = 1/\varepsilon_2 = \varepsilon$ — this amounts to assuming a single relative sectoral shock. Then, using (1.2) and data for each sector's output and labor input, the aggregate and sectoral Solow residuals are easy to calculate.[8] By doing

[6]Labor's share of income for sector i, or α_i, was computed from the formula shown below using data from the National Income and Product Accounts:

$$\alpha_i = \frac{COM_i}{NI_i + CCA_i - PI_i},$$

where COM_i is Compensation of Employees for sector i, NI_i is National Income, CCA_i is the Capital Consumption Allowance, and PI_i is Proprietor's Income.

[7]The adjustment costs are quantitatively trivial in magnitude. The loss in labor input due to adjustment costs averages less than 0.0028 percent of total employment in the simulations undertaken.

[8]The assumption on the functional form for the sectoral disturbances allows them to be easily identified.

this it is found that the aggregate shock has a percentage standard deviation of 0.04 and a serial correlation coefficient of 0.93. The numbers for the sectoral shock are 0.015 and 0.93.

The aggregate and sectoral shocks are two-state Markov processes: $z_t \in Z = \{\exp^{\xi}, \exp^{-\xi}\}$ with $\Pr[z' = z_1 |\ z = z_1] = \Pr[z' = z_2 |\ z = z_2]$, and $\varepsilon \in E = \{\exp^{\zeta}, \exp^{-\zeta}\}$ with $\Pr[\varepsilon' = \varepsilon_1 |\ \varepsilon = \varepsilon_1] = \Pr[\varepsilon' = \varepsilon_2 |\ \varepsilon = \varepsilon_2]$. The parameters ξ and ζ are chosen so that the time series properties for the aggregate and sectoral disturbances in the model inherit the time series behavior of the aggregate and sectoral Solow residuals. This implies setting $\xi = .04$, $\Pr[z' = z_1 | z = z_1] = .965$, $\zeta = .015$ and $\Pr[\varepsilon' = \varepsilon_1 | \varepsilon = \varepsilon_1] = .965$.[9]

1.3.5 Investment

Finally, in the U.S. economy consumption is relatively smooth, and investment is procyclical and highly volatile. This motivates subtracting a certain amount of output, equal to investment, from the right hand side of the resource constraints.[10] The function $I_i(z, \varepsilon_i)$ is intended to capture this. Let the investment functions, $I_i(z, \varepsilon_i)$,

[9]It is straightforward to calculate that the percentage standard deviations of the aggregate and sectoral disturbances are given by ξ and ζ. Likewise, the formulae for autocorrelation coefficients for the shocks are $2\Pr[z' = z_1 | z = z_1] - 1$ and $2\Pr[\epsilon' = \epsilon_1 | \epsilon = \epsilon_1] - 1$, respectively.

[10]The aggregate disturbance will not affect the solution to the model if there is no investment term in the resource constraint (1.6). This is immediate from problem P(2). Without the $I_i(z, \varepsilon_i)$ term, it is easy to see that z can be factored out of the first term on the righthand side of P(2). Hence it can't affect the maximization.

have the form

$$I_i(z,\varepsilon_i) = \begin{cases} e^{\sigma+\sigma_i} I_i^*, & \text{if } z = e^{\xi} \text{ and } \varepsilon_i = e^{\zeta}, \\ e^{\sigma-\sigma_i} I_i^*, & \text{if } z = e^{\xi} \text{ and } \varepsilon_i = e^{-\zeta}, \\ e^{-\sigma+\sigma_i} I_i^*, & \text{if } z = e^{-\xi} \text{ and } \varepsilon_i = e^{\zeta}, \\ e^{-\sigma-\sigma_i} I_i^*, & \text{if } z = e^{-\xi} \text{ and } \varepsilon_i = e^{-\zeta}, \end{cases}$$

where the means and standard deviations of $\ln I_i(z,\varepsilon_i)$ are given by $\ln I_i^*$ and $\sqrt{\sigma^2+\sigma_i^2}$.

In the U.S., aggregate investment is approximately 20 percent of GNP. This implies that in the model steady state $I_1 + pI_2 = .2[y_1 + py_2]$, where p is the relative price of good two. Also, the goods producing sector generates two-thirds as much output as the service sector. If it is assumed that investment spending is spread across sectors proportionally, then the models steady state should display the feature that $I_1/I_2 = y_1/y_2$. Assuming this, along with $I_1 + pI_2 = .2[y_1 + py_2]$, implies $I_1^* = .0378$ and $I_2^* = .0605$. In the U.S. data, investment is four times as volatile as output and the correlation coefficient between aggregate investment and output is 0.95. The percentage standard deviations for the investments were chosen to mimic these observed facts. This involved setting $\sigma = .08$, $\sigma_1 = .06$, and $\sigma_2 = .08$.

1.4 Findings

The cyclical properties of the above model are developed through simulation. As is now standard, the procedure is to compare a set of stylized facts characterizing the business cycle behavior of the model with a analogous set describing U.S. postwar business cycle behavior over the 1964.1-1987.4 sample period. Appendix A details the computational procedure used to calculate the decision-rules associated with the planner's problem. The procedure used to compute the decision-rules is complicated by the presence of the inequality constraint (1.8). With these decision-rules in hand, 200 samples of 96 observations (the number of quarters in the U.S. sample period) are simulated. Each simulation run corresponds to a randomly generated sample of 96 realizations of the z and ε processes. The data from the simulations is logged (where applicable) and H-P filtered, as is the data for the U.S. economy, and average moments over the 200 samples are computed for each variable of interest.

1.4.1 Impulse-Response Functions

The dynamic effects that aggregate and sectoral disturbances have on sectoral employment and aggregate nonemployment can be represented in terms of impulse-response functions.[11] This is done by fitting a first-order vector autoregression of the form $\pi' = c + b\pi + \nu$ to the simulated data, where $\pi = [\pi_1, \pi_2, \pi_3]^T$, c and b are 3×1 and 3×3 parameter vectors, and ν is a 3×1 vector of approximation errors. Figure 2 plots the impulse response functions associated with an aggregate shock, where the economy is assumed to be in a steady state initially. Employment in both sectors rises, while aggregate nonemployment (or $1 - \pi_1 - \pi_2$) falls. Notice that it takes the

[11] To be nonemployed is defined here as not working. In the model the number of agents who are nonemployed is $1 - \pi_1 - \pi_2 - \pi_3$. This is an exact concept and does not match up precisely with the notion of being unemployed. In the U.S. data an agent is counted as being unemployed if he is not working, but has looked for a job within the last four weeks.

Figure 1.2. Impulse Responses: Aggregate Shock

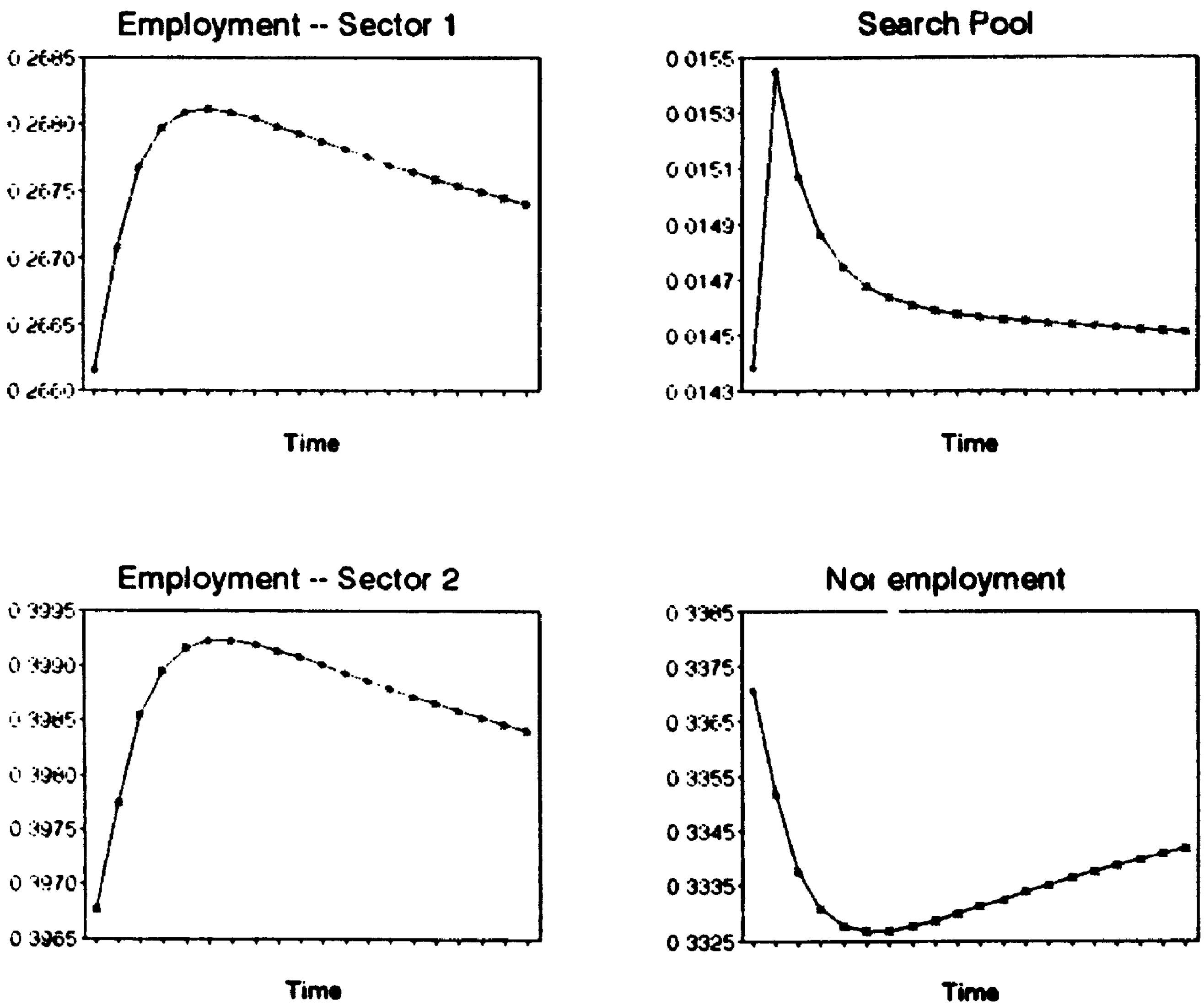
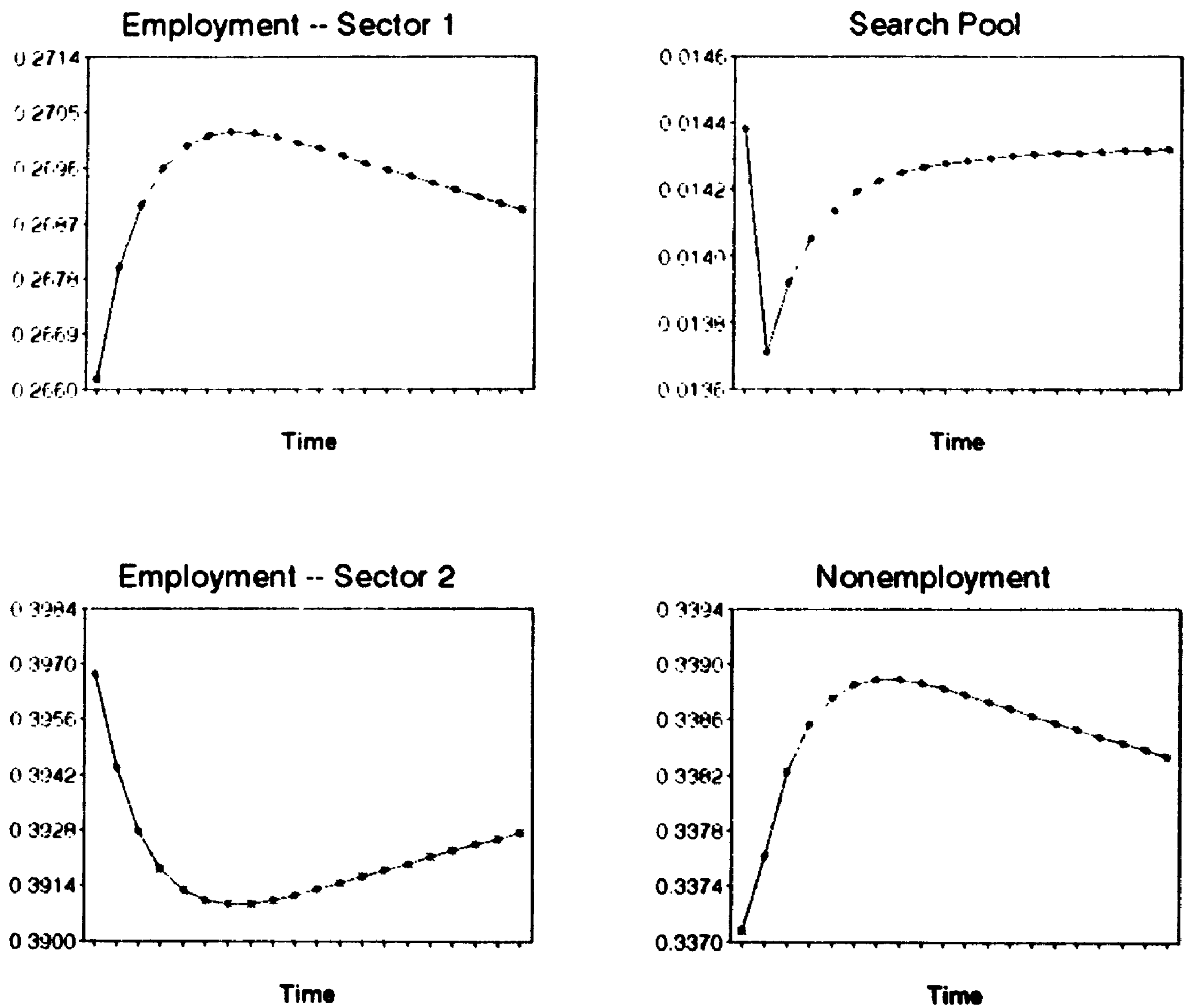


Figure 1.3. Impulse Responses: Sectoral Shock



economy five periods to move agents out of the searching pool and home sector into work in the two market sectors. This illustrates the influence of adding the search pool to the model. The results here are consistent with Jovanovic's (1987) argument that a positive serially correlated aggregate shock will simultaneously increase sectoral employments and search, and decrease aggregate nonemployment. Similarly, Figure 3 shows the impulse response functions for a sectoral shock. A positive sectoral shock increases the productivity of the goods sector relative to services. Consequently, employment in goods (services) production rises (falls). Again, it takes the economy about five to six periods to go through the adjustment process. Observe that nonemployment rises following the sectoral shock. This transpires since sector two is larger than sector one; more workers are withdrawn from sector two in response to the technology shock than are added to sector one with the difference leaving the labor force.

1.4.2 Aggregate and Sectoral Fluctuations

The amount of job creation in sector i during period t is given by $\max\{0, \pi_{i,t} - \pi_{i,t-1}\}$. Thus, the sector-$i$ job creation rate is defined to be $\max\{0, \pi_{i,t} - \pi_{i,t-1}\}/\pi_{i,t-1}$. Likewise, for sector-$i$ the job destruction rate is $\max\{\pi_{i,t-1} - \pi_{i,t}, 0\}/\pi_{i,t-1}$. The sum of these job creation and destruction rates defines the sector-i job reallocation rate. It follows that the aggregate job creation and destruction rates are $\sum_{i=1}^{2} \max\{0, \pi_{i,t} - \pi_{i,t-1}\}/\sum_{i=1}^{2} \pi_{i,t-1}$and $\sum_{i=1}^{2} \max\{0, \pi_{i,t-1} - \pi_{i,t}\}/\sum_{i=1}^{2} \pi_{i,t-1}$. The (weighted) sum of the aggregate job creation and destruction rates defines the aggregate job reallocation rate.

Descriptive statistics characterizing the cyclical behavior of U.S. labor market aggregates are presented in Table 1. Table 2 presents the same statistics for the model. The model reproduces the cyclical pattern of job creation, destruction and reallocation displayed in the U.S. data relatively accurately. Specifically,

- In both the model and the data, the job creation rate moves procyclically while the job destruction and reallocation rates are countercyclical. The correlations between these variables, on the one hand, and GNP and employment, on the other, are also close to those found in U.S. data.
- In both the model and the data the job destruction rate is more volatile than either the job creation or reallocation rates. This reflects the importance of the asymmetric nature of the employment process. It is much easier to fire people than to hire them.
- In the data the correlation between hours and productivity is low, as evidenced by the correlation coefficient of 0.20. For the model the number is 0.81, which is too high. On this dimension the model performs more or less the same as the standard model with indivisible labor, but worse than models that include government spending or household production — see Hansen and Wright (1992). Another shortcoming is that hours worked in the model is much less volatile than in the data. Consequently,

Table 1.1: Cyclical Behavior of U.S. Labor Market Aggregates

Quarterly, 1964.1-1987.4

VARIABLES	S.D.(%)	CORR/OUTPUT	CORR/EMPLOYMENT
OUTPUT	2.50	1.00	0.86
EMPLOYMENT	1.66	0.86	1.00
HOURS	1.96	0.91	0.98
NONEMPLOYMENT		-0.93	-0.92
JOB CREATION RATE	0.71	0.46	0.12
JOB DESTRUCTION RATE	1.82	-0.47	-0.20
JOB REALLOCATION RATE	0.56	-0.14	-0.12
PRODUCTIVITY	1.06	0.66	0.20

NOTE The U S economy analyzed in this paper consists of two sectors One of them, Sector 1, is the Goods sector which includes three 1-digit SIC(1987) industries Mining, Construction and Manufacturing The other, Sector 2, is the Service sector which includes the following SIC industries. Transportation and Public Utilities, Wholesale Trade and Retail Trade, Finance-Insurance and Real Estate, and Services All industry time series are taken from CITIBASE(1989) Since quarterly GNP by industry is not available, National Income is used as substitute The availability of data on average weekly hours worked by industry determines the sample periods starting from 1964.1 to 1987.4.
For productivity, CORR/EMPLOYMENT represents the correlation of productivity and hours. All series are logged (where applicable) and detrended using the Hodrick-Prescott filter.

Table 1.2: Cyclical Behavior of Labor Market Aggregates

Model: 200 Simulations with 96 Observations Each

VARIABLES	S.D.(%)	CORR/OUTPUT	CORR/EMPLOYMENT
OUTPUT	1.92	1.00	0.87
EMPLOYMENT	0.29	0.87	1.00
HOURS	0.29	0.87	0.99
NONEMPLOYMENT		-0.87	-1.00
JOB CREATION RATE	1.69	0.26	0.03
JOB DESTRUCTION RATE	1.95	-0.30	-0.22
JOB REALLOCATION RATE	1.54	-0.04	-0.12
PRODUCTIVITY	1.67	0.99	0.61

NOTE: All time series are logged (where applicable) and detrended using the H-P filter. The statistics shown in all tables are the average values after 200 simulations of 96 observations each. For productivity, CORR/EMPLOYMENT represents the correlation of productivity and hours.

Table 1.3: Cyclical Behavior of Sector Labor Aggregates in U.S. Economy

Quarterly, 1964.1-1987.4

VARIABLES	S.D.(%)		COR with OUTPUT		CORR with EMPL'	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
OUTPUT	4.12	1.50	1.00	1.00	0.87	0.77
EMPLOYMENT	2.94	1.00	0.88	0.77	1.00	1.00
HOURS	3.50	1.02	0.93	0.81	0.98	0.98
JOB CREATION RATE	1.31	0.56	0.32	0.47	0.03	0.14
JOB DESTRUCTION RATE	1.68	2.95	-0.49	-0.40	-0.24	-0.16
JOB REALLOCATION RATE	0.96	0.47	-0.37	0.31	-0.24	0.07
PRODUCTIVITY	1.54	0.90	0.56	0.75	0.12	0.18

NOTE For productivity CORR/EMPLOYMENT represents the correlation of productivity and hours

Table 1.4: Cyclical Behavior of Sector Labor Aggregates

Model: 200 Simulations with 96 Observations Each

VARIABLES	S.D.(%)		COR with OUTPUT		CORR with EMPL'	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
OUTPUT	2 24	2 16	1 00	1 00	0 76	0 75
EMPLOYMENT	0 68	0 63	0 76	0 75	1 00	1 00
HOURS	0 68	0 63	0 76	0 75	1 00	1 00
JOB CREATION RATE	2 65	2 15	0.30	0.37	0.10	0 18
JOB DESTRUCTION RATE	2 97	2 32	-0.37	-0.38	-0.22	-0.19
JOB REALLOCATION RATE	1 85	1 44	-0.08	-0.02	-0.10	0 00
PRODUCTIVITY	1 77	1 74	0 95	0 96	0.57	0 57

Table 1.5: Relation Between Job Creation & Destruction

Correlation

	Aggregate	Sector 1	Sector 2
U.S. Data: (Quarterly, 1964.1-1987.4)	-0.50	-0.37	-0.44
Model (200 Simulations)	0.40	-0.13	-0.18

productivity fluctuates more in the model than in the data. This is due to the presence of adjustment costs for hiring labor.[12]

Next, some stylized facts describing the behavior of U.S. labor market variables at the sector level are given in Table 3. Table 4 presents the same set of facts for the model. The key findings here are:

- In the data, the job creation, destruction and reallocation rates display the same pattern of cyclical behavior at the sectoral level as they do for the economy as a whole. There is, however, one exception: while the job reallocation moves countercyclically in the goods producing sector it moves procyclically in services.[13] The model replicates fairly closely the correlation structure between these variables and output, except for the procyclical movement of the job reallocation rate in the service sector.
- The model and data share the feature that output and employment are more volatile in goods production than in services.
- The model does a much better job matching the hours/productivity correlations observed at the sectoral level.

Finally, Table 5 reports negative correlations between job creation and destruction

[12]Hours fluctuates more than productivity in the data. Hansen (1985) matched this fact by introducing indivisible labor into an otherwise standard stochastic growth model. In the current analysis productivity is more volatile than hours, notwithstanding the use of indivisible labor.

[13]The size of the service sector has increased over time while the volume of goods production has declined. Jobs created in the service sector may accelerate during booms and jobs destroyed in the goods sector may speed up in recessions. This hypothesis is consistent with findings in Loungani and Rogerson (1989).

rates, at both the aggregate and sectoral levels. Similar findings are reported in Mortensen (1994). On this,

- The model yields mixed results here. On the one hand, a positive correlation between aggregate job creation and destruction is displayed by the model. On the other, the model does replicate the negative association between job creation and destruction observed at the sectoral level, although it understates the size of the correlations.

1.4.3 The Determination of Aggregate Unemployment

How much of unemployment can be accounted for by aggregate and sectoral shocks? In the absence of technology shocks there would be no steady-state search unemployment in the model. Thus, the average value for π_3 is a measure of the amount of unemployment due to aggregate and sectoral disturbances.[14] On this account 1.20 percent of the labor force is unemployed. To get a rough estimate of how the aggregate and sectoral shocks contribute to unemployment, the aggregate and sectoral shocks can be shut down in turn. When the aggregate shock is shut down (i.e., $\xi = 0$ and $\zeta = .015$.) the average value for of π_3 falls to 0.97. The average value of π_3 drops to 1.00 when the sectoral shock is turned off (i.e., $\xi = 0.04$ and $\zeta = 0$). Thus, aggregate and sectoral shocks have a similar effect on the average level unemployment. Finally, the procyclical nature of quits in the U.S. economy suggests

[14]It is being assumed that all agents in the search pool would qualify as being unemployed, as measured in the U.S. data — see footnote 11.

that job search is procyclical (see Jovanovic (1987)). The model predicts that the search is procyclical, that is the correlation between π_3 and output is 0.61.

1.4.4 Discussion

The job creation and destruction rates computed above represent the lower bounds on the amount of job creation and destruction in the U.S. economy. To ease the burden of the quantitative analysis the economy was dichotomized into two broad sectors, goods and services. If the economy was disaggregated down further into many sectors the amount of job creation and destruction would increase.[15] In fact, the amount of job creation, destruction, and reallocation could be disaggregated down to the level of the plant, as Davis and Haltiwanger (1990) do for the manufacturing sector of the U.S. economy. They find that job creation is procyclical, job destruction is countercyclical, and the latter is more volatile than the former. In a model with many sectors, and perhaps many plants within a sector, the amount of steady-state search unemployment due sectoral and plant-specific shocks should increase. On this, in a study of 26 U.S. industries, Loungani and Rogerson (1989) find that approximately 5.5 percentage points of unemployment among workers can be accounted for by industry switchers.

[15] The job creation rate in an N-sector model is given by $\sum_{i=1}^{N} \max\{0, \pi_{i,t} - \pi_{i,t-1}\} / \sum_{i=1}^{N} \pi_{i,t-1}$. Now, consider aggregating the N sectors up into 2 sectors. The rate of job creation for the aggregated 2-sector model would be $\max\{0, \sum_{i=1}^{M} (\pi_{i,t} - \pi_{i,t-1})\} / \sum_{i=1}^{N} \pi_{i,t-1} + \max\{0, \sum_{i=M+1}^{N} (\pi_{i,t} - \pi_{i,t-1})\} / \sum_{i=1}^{N} \pi_{i,t-1}$. Clearly, the latter sum is smaller than the former one.

1.5 Concluding Remarks

A multisector dynamic general equilibrium model is constructed here to analyze the cyclical pattern of job creation and destruction. The two main ingredients in the model are the Lucas-Prescott (1974) idea that it takes time to find employment and the Rogerson (1988)/Hansen (1985) notion of indivisible labor. It is found that the model can successfully replicate the cyclical patterns of job creation, destruction and reallocation that is observed at both the aggregate and sectoral levels in the U.S. economy. Specifically, job creation rates move procyclically in the model while job destruction rates move countercyclically, as they do in they data. Also, in the model job destruction is more volatile than either job creation or reallocation, a feature displayed in the data. Finally, it is found that aggregate and sectoral disturbances contribute non-negligibly to unemployment.

In the model presented here workers were assigned their employment status via a lottery. They were perfectly insured against the possibility of dismissal, in the sense that their consumption in a period was not contingent upon their employment status. One can imagine a world where no such insurance exists. Suppose, instead, that individuals can only insure themselves by saving in the form of a simple asset, such as money or government bonds. Each period those agents currently working in a sector decide whether to stay at work, enter the unemployment pool to search for a new job in another sector, or leave the labor force. Agents in the unemployment pool decide whether to take a job in some sector, remain in the unemployment pool for another period, or leave the labor force. Likewise, those individuals at home must decide whether or not to enter the labor force. Clearly, an individual's decision will be predicated upon both his idiosyncratic circumstance (asset holdings, employment status) and the aggregate situation (the distribution of agents and state of technology in each sector). While computationally more complicated, such an analysis would undoubtedly share many of the features of the above model. But it would permit a much richer analysis along some dimensions. For instance, one could study the effect that government policies, such as unemployment insurance, have on intersector mobility and unemployment.[16] The current analysis can be viewed as a first step toward such a model.

[16]This policy experiment could be viewed as embedding the analysis of Hansen and Imrohoroglu (1992) into a multisector general equilibrium model of the form presented here.

APPENDIX I

Computation

Modified Discrete State Space Approach With Value Function Approximation

The neoclassical growth model ca. be solved using standard discrete state space dynamic programming techniques. In economies with multiple sectors or multiple agents, the standard approach becomes unworkable due to the curse of dimensionality, which limits the practicability of standard discrete state space dynamic programming techniques for large problems.

An alternative treatment of the problem is to store a limited set of coefficients characterizing a parameterized value function and momentary return function.[17] The parameterized objective function can then be maximized using an optimization routine. Two benefits derive from this method: First, computation costs are reduced dramatically; and second, the maximizers are no longer constrained to lie in a discrete subset set of the constraint set.

An obvious candidate in the family of simple functions to use to approximate more complicated functions is the polynomial. However, there are two problems

[17] A discussion of numerical techniques used to solve dynamic equilibrium models can be found in Danthine and Donaldson (1995).

associated with polynomial approximation. First, practical concerns prevent using high order polynomials (even given the Weierstrass theorem). Second, the adequacy of polynomial approximations depends on the differentiability properties of the function that is being approximated. Often, for a smooth function a lower degree polynomial can be used.[18]

The representative agent's optimization problem, characterized by problem P(2) in Section 2, can be simplified to one with only linear constraints by using the following lemma. For this simplified problem, it is easy to check the convexity of the constraint set.

Lemma 1: *The transition constraint*

$$\sum_{i=1}^{2} \max\{0, \pi_i' - \pi_i\} \leq \pi_3 \tag{1.10}$$

is equivalent to following set of linear inequality constraints:

$$\pi_1' + \pi_2' \leq \pi_1 + \pi_2 + \pi_3, \tag{1.11}$$

$$\pi_1' \leq \pi_1 + \pi_3, \tag{1.12}$$

$$\pi_2' \leq \pi_2 + \pi_3. \tag{1.13}$$

[18] Given the assumptions placed on tastes and technology here, the value function will be strictly increasing, strictly concave and continuously differentiable (Stokey et al (1989), Chap 9).

Proof: It is trivial to verify that the set constrained by (1.10) is same as the one constrained by (1.11), (1.12) and (1.13) . If (1.10) holds, then the following must be true,

$$(\pi_1' - \pi_1) + (\pi_2' - \pi_2) \leq \pi_3, \tag{1.14}$$

$$\pi_1' - \pi_1 \leq \pi_3, \tag{1.15}$$

$$\pi_2' - \pi_2 \leq \pi_3. \tag{1.16}$$

But this is merely (1.11)– (1.13). On the other hand, from (1.14) – (1.16) it is easy to derive that

$$\max\{0, \pi_1' - \pi_1\} + \max\{0, \pi_2' - \pi_2\} \leq \max\{0, \pi_3\}, \tag{1.17}$$

which is equivalent to the transition constraint (1.10). □

Let $\mathcal{F}$ represent the space of continuous, bounded functions and consider the mapping $T : \mathcal{F} \rightarrow \mathcal{F}$ defined by P(3).

$$\begin{aligned} V^{j+1}(\pi_1, \pi_2, \pi_3; z, \varepsilon_1, \varepsilon_2) \quad = \quad & \max_{\{\pi_1', \pi_2', \pi_3'\}}\{\tfrac{A}{\rho} \ln(\sum_{i=1}^{2} \theta_i(z\varepsilon_i w^{\alpha_i}(\pi_i' - \\ & \gamma_i(\max[\pi_i' - \pi_i, 0])^{\lambda_i})^{\alpha_i} - I_i(z, \varepsilon_i))^{\rho}) \\ & +(1 - A)[\pi_3' \ln(1 - s) + (\sum_{i=1}^{2} \pi_i') \ln(1 - w)] \\ & +\beta E[V^j(\pi_1', \pi_2', \pi_3'; z', \varepsilon_1', \varepsilon_2') | \pi_1, \pi_2, \pi_3; z, \varepsilon_1, \varepsilon_2]\}, \end{aligned} \tag{$P(3)$}$$

subject to the constraints (1.11) – (1.13) and

$$\sum_{i=1}^{3} \pi_i' \leq 1, \quad \pi_i' \geq 0. \tag{1.18}$$

The mapping T maps V^j to V^{j+1}. This operator is a contraction mapping that has as its unique fixed point the function V defined by P(2).[19] This last observation motivates the computational procedure used here consisting of the following steps:

1. A grid is defined over the model's state space. Specifically, it is assumed that $\pi_1 \in [.243, .297]$, $\pi_2 \in [.360, .440]$, and $\pi_3 \in [0, .024]$.[20] Three grids of 13 equally spaced points are layered over these intervals. These sets of grid points are denoted by Π_1, Π_2, and Π_3, respectively.

2. An initial guess for the 2nd degree polynomial used to approximate the value function over this grid is made.

3. Given the guess for the value function, a maximization routine is used to solve the constrained nonlinear optimization problem P(3) for the optimal decision-rules.[21] This is done for each of the 8,788 points in the set $\Pi_1 \times \Pi_2 \times \Pi_3 \times Z \times E$.

4. Using the solution obtained for the optimal decision-rules, a revised guess for the value function is computed. This is done by choosing a new 2nd degree

[19] It is trivial to check that P(3) satisfies Blackwell's sufficiency conditions for a contraction mapping — see Stokey et al (1989).

[20] By simulating the model it was determined that system never left these intervals.

[21] This was done using M.J.D. Powell's GETMIN subroutine developed for solving constrained nonlinear optimization problems.

polynomial to approximate the value function. In particular, from P(3) a value for V can be computed for each grid points in the set $\Pi_1 \times \Pi_2 \times Z \times E$. A 2nd degree polynomial is then fitted to these points via least squares.

5. The decision-rules are checked for convergence.

Once the decision-rules have been obtained, the model can be simulated and various statistics are generated consequently. Note that function values for the decision-rules will have been computed for each point in the set $\Pi_1 \times \Pi_2 \times \Pi_3 \times Z \times E$. It is then easy to obtain values for the decision-rules at any point in the space $[.243, .297] \times [.360, .440] \times [0, .024] \times Z \times E$ by using multilinear interpolation – see Press et al [18]. The adequacy of using a 2nd degree polynomial to approximate the value function can be assessed from a R^2 statistic. The R^2 obtained from using the 2nd degree polynomial was 0.96. Additionally, one could fit a higher order polynomial to the values of V obtained in the grid. Using some appropriate metric, the distance between this polynomial and the 2nd order one can be computed over some desired space. For instance, using the standard Euclidean norm the distance between a 2nd and 3rd degree polynomial was 0.022 when evaluated at some 70,304 points along a mesh spanning the state space.[22] The mesh was constructed by making the original grid twice as fine. While the third degree polynomial fit better (it had an R^2 of 0.99) it involved more computer time without any noticeable change in the

[22] The average value of $|V|$ over the original grid is about 16.0. Thus, the distance is fairly small in relative terms.

results.

The Data Set

As described in Figure 1, the goods-producing and service-producing sectors are made up by seven SIC one-digit industries: Mining (1), Construction (2) , Manufacturing (3), Transportation and Public Utilities (4), Wholesale Trade and Retail Trade (5), Finance, Insurance and Real Estate (6) and Services (7). Here the goods-producing sector includes the first three industries while the rest make up the service sector.

All the time series for the postwar U.S. economy are obtained from Citibase. Exceptions are the series for noncorporate capital consumption allowance by industry which came from the National Income and Product Accounts. Output for industry i is measured in 1982 prices. Total hours worked in industry i is the product of employment and the weekly hours worked per employee in that industry. The output, employment, hours and unemployment series are deflated by the civilian population. Citibase codes are contained in Table 6.

Table 1.6: Codes of the Time Series in Citibase

Sample Period: 1964.1-1987.4

VARIABLES	INDUSTRIES						
	1	2	3	4	5	6	7
A: OUTPUT							
National Income	GYWM	GYWC	GYM	GYWTU	GYNRW+GYNRR	GYFIR	GYS
GNP (82)	GA8G14	GA8G15	GA8GM	GA8GTU	GA8GW+GA8GR	GA8GFE	GA8GS
GNP	GAG14	GAG15	GAGM	GAGTU	GAGW+GAGR	GAGFE	GAGS
B: LABOR							
Employment	LPMI	LPCC	LPEM	LPTU	LPT	LPFR	LPS
Weekly Hours Worked Per Employee	LWMI6	LWCC	LPHRM	LWTU	LWTWR	LWFR6	LWS
Unemployment Rate	LURMI	LURC	LURM	LURTPU	LURWR	LURFS	LURFS
C: LABOR SHARE							
Compensation of Total Employees	GAPMI	GAPCC	GAPM	GAPTPU	GAPW+GAPR	GAPFF	GAPS
Proprietor's Income	GAYPMI	GAYPCC	GAYPM	GAYPTU	GAYPTW+GAYPRT	GAYPF	GAYPS
Corporate Capital Consumption Allowance	GACMI	GACCC	GACM	GACTPU	GACW+GACR	GACFF	GACS
D: Population							
Civilian Population				PO16			

NOTE - The industry numbers are defined in the text of Appendix B
- All time series, except ones at annual frequency, are seasonally adjusted
- All time series in groups A and C are nominal, except GNP which is in 1982 dollars
- All series in groups A and C are annual except national income which is quarterly, and all series in groups B and D are monthly

Chapter 2

Unemployment Insurance Analysis in a Search Economy

2.1 Introduction

Most of the previous research on unemployment insurance (UI) has focused on the costs of the program, such as increasing unemployment duration. The other side of the coin is the benefits that arise from UI programs. The question to policy makers is, of course: Do the benefits of existing UI programs outweigh the costs? [See Baily (1978).]

The benefit issues of a UI program can be addressed along different dimensions. For instance, researchers have found that UI as insurance for workers can help workers to smooth their consumption during job transition. [See Baily (1977) and Gruber

(1994), whose empirical results show that a 10 percentage point increase in the UI replacement rate can reduce the fall in consumption due to unemployment by 2.7%.] Furthermore, UI can improve social welfare, but it might be harmful to the economy if the moral hazard is severe. [See Hansen and Imrohoroglu (1992).] In addition, welfare states can co-exist with a low unemployment rates when liberal unemployment insurance is financed by progressive income taxation. [See Ljungqvist and Sargent (1994).][1] However, there are other dimensions to which macroeconomists have not paid close attention. For example, UI subsidizes job searches so that the process of searching for jobs is prolonged, and consequently, the average wage received by those just exiting the unemployment pool may be increased in the model economy. This potential benefit of a UI program will be explored in this paper. As pointed out by Gruber (1994), the public provision of unemployment insurance may raise welfare by filling the missing market for a state-contingent payment. The optimal size of the program will trade off those consumption smoothing and job search gains against the induced distortions of individual and firm behavior.

As in Hansen and Imrohoroglu (1992), this paper employs a quantitative dynamic general equilibrium approach to address the benefit issue of a UI program. In contrast to Hansen and Imrohoroglu (1992),this research is done in a search economy. A precautionary saving model has been developed to explore how UI helps job searchers to smooth their consumption. This essay also examines the impact of unemploy-

[1] Their paper is motivated by the above phenomenon observed in Sweden. As a matter of fact, as reported in the July 29th 1995 issue of *The Economist*, the current Swedish unemployment rate is as high as 8.2 percent.

ment insurance on the average level of unemployment, duration of unemployment, and the average wage rate receive by workers who exit unemployment. The welfare consequences of implementing certain policies related to an unemployment insurance program are also analyzed.[2] In a related work, Andolfatto and Gomme (1995) have employed general equilibrium methodology to evaluate the unemployment insurance programs proposed in Canada before and after the 1972 reform.

The current search model uses a simple one-firm/one-worker setup.[3] Each agent's production site is subject to an idiosyncratic technological shock[4]. Risk averse agents choose to work at a current level of productivity or search for a new level so as to maximize their lifetime expected utility[5]. It is assumed that work and search are mutually exclusive. Thus, job searchers comprise the unemployed in the model economy. Agents are not allowed to borrow from each other for consumption purposes[6]. Agents own different amounts of assets, and they save for both consumption smoothing and investment purposes. This imperfect capital market assumption naturally

[2]The dynamic equilibrium approach has been also employed by a few other researchers on the issue of optimal unemployment insurance. Hopenhayn and Nicolini (1995) have analyzed the optimal insurance contract between a risk neutral principal and a risk averse agent who faces job uncertainty in a partial equilibrium setup. Similar work has been done in a dynamic contract model with private information by Wang and Williamson (1995). How to design an optimal insurance contract which overcomes the moral hazard is the main focus of their research.

[3]A full analysis integrating the worker and firm sides of the picture to show if the results presented here should be modified is a desirable extension of this research. The main difficulty related to the integration is that the model needs to keep track of another distribution, that is, the distribution of agents across firms. Given that this paper focuses on an individual agent's behavior, not much benefit can be derived from this more complex economy.

[4]In Hansen and Imrohoroglu (1992), all workers receive same amount of income; in this model, workers receive different amounts of income from work if they work at different levels of productivity.

[5]Although the model evaluates leisure, it only allows agents to choose between work and search.

[6]The assumption of capital market imperfection is consistent with the fact that many low income households in U.S. face a borrowing constraint.

leaves room for an unemployment insurance program. In addition, apart from UI, the government provides a lump-sum transfer to each agent. Both UI benefits and lump-sum transfers are financed by income taxes, and the UI benefit is endogenously determined in the economy. The taxes affect agents' decisions to work, search, save and invest. Finally, the stationary search equilibrium analyzed here is similar to the sectoral demand-shift model originally proposed by Lucas and Prescott (1974) and further developed by Jovanovic (1987)[7].

The quantitative analysis presented here reveals that unemployment insurance improves social welfare as the model economy moves from a no-welfare state to a welfare state in the benchmark setup. As expected, UI helps to prevent consumption from falling for unemployed workers. Therefore, UI plays a role as insurance for workers. The model also predicts that a more generous program will cause the unemployed to search longer so that the average wage received by the agents exiting unemployment is increased. However, a more generous program does not necessarily lead to a higher unemployment rate in an income tax regime. In particular, income tax directly affects job search by decreasing all agents' income and indirectly discourages job search by increasing before tax interest rate, which reduces average wage income further.

The rest of this paper is structured as follows. Section 2 describes a basic economic environment and optimization problems for both workers and searchers. A stationary recursive competitive equilibrium is also defined in this section. Section 3 calibrates

[7] In absence of a physically described process of search, Lucas and Prescott (1974) proved that without aggregate uncertainty there exists a unique stationary equilibrium in their island economy.

a benchmark model. Section 4 examines the effects of UI programs under different scenarios, and the possible extensions of this model are presented in Section 5. Finally, the solution algorithm is characterized, and the measurement of the variables is delivered in Appendix II.

2.2 The Basic Model

2.2.1 The Economic Environment

There is a continuum of agents with measure one in the economy. An agent can spend his time either working, searching for new production opportunities, or enjoying leisure. Each agent has his own production site, which is subject to idiosyncratic shock. Production occurs before consumption.

In each period, an agent can work at his site at the current productivity level or abandon it to search for new production opportunities. The abandonment of existing productivity sites by workers can be interpreted as a job separation, that is, a worker can quit the position due to its poor employment prospects. Therefore, the model is not silent on moves from employment into unemployment. Simultaneously, the unemployed are looking for new production opportunities, which can be modeled as a job searching process. Therefore, the model also considers moves from unemployment into employment.

As in Hansen and Imrohoroglu (1992), agents are not allowed to borrow from each other for consumption purposes, and there is no private insurance available to agents in the economy[8]. In the absence of an unemployment insurance program, agents have to self-insure themselves against potential downturns in the future. Therefore, unemployment insurance plays a role in an economy with an imperfect capital market.

In the presence of a UI program, all unemployed agents are assumed to be eligible for the insurance benefit, b_t. For simplicity, UI is assumed to be distributed in a lump-sum fashion to unemployed workers. The value of the average UI benefit, b_t, is equal to the product of the replacement ratio, $\rho \in (0,1)$, and average after-tax wage income[9]. This implies that the amount of UI collected by an agent is independent of his previous wage income. However, UI is less attractive and consequently has less influence on workers with a higher wage income. This is because they incur a higher opportunity cost of searching.

Apart from unemployment insurance, the government provides an amount μ_t as a lump-sum transfer to each agent. This construct is consistent with the observation that part of tax revenue is used to finance social programs other than UI. In the model economy, both UI and lump-sum transfer programs are financed by income taxes. For simplification, the model assumes that all income is taxed at the rate τ.

[8] One could assume that idiosyncratic shock, z, is private information. This would work to exclude the private provision of insurance.

[9] According to the conventional definition, the replacement ratio is the ratio of net unemployment benefit to average after-tax income from work. Another definition is the ratio of gross UI benefit to gross wage. The first is chosen here because benefits and taxes are two different policy parameters. It is desirable to isolate their effects on individual behavior, which are the focus of the current paper. What matters for an unemployed individual is the ratio of net income out of work to net income in work. The first definition makes this ratio independent of the tax parameters.

If an agent chooses to operate his firm in period t, he supplies l_t units of his time and employs k_t units of capital in production. The production function is assumed to have the Cobb-Douglas form, $zk_t{}^{\alpha}l_t^{1-\alpha}$. The capital stock, k_t, equals his own capital goods, a_t, plus the capital stock, $k_t - a_t$, rented from others at market rental rate, r_t. It is assumed that the capital depreciates at the rate of δ. A new production opportunity arrives following a first-order Markov transition law,

$$G(z', z) = \Pr(z_{t+1} \leq z' \mid z_t = z),\ z', z \in Z.$$

If an agent decides to search for a new production opportunity, he will be unemployed in period t. Each searcher has to supply a fixed amount of effort, e, for getting a draw of new production opportunities, z_{t+1}. The variable, z_{t+1}, is distributed over the set $Z \equiv \{z_l, z_m, z_h\}$ with mean, $\bar{z}$, and standard deviation, σ. At each point in time, the distribution over Z is common knowledge to agents. An unemployed worker can randomly draw a new production opportunity only once a period. To simplify the problem further, this model assumes that all unemployed agents face the same distribution of z_{t+1}, $\Phi(z_{t+1})$.

In each period, each agent is indexed by his idiosyncratic shock, $z_t \in Z$, asset holdings, $a_t \in A = [0, \overline{A}]$, and labor market status, $s_t \in \{0, 1\}$. The variable, s_t, takes the value 1, if an agent chooses to produce in period t, 0 otherwise. At the beginning of period t, both employed and unemployed agents observe their period-t production opportunity. Based on his wealth, for a given interest rate, r_t, a typical agent makes his consumption, asset holdings, and working/searching decisions to maximize his lifetime expected utility[10],

$$E\sum_{t=0}^{\infty}\beta^t U\left(c_t, s_t l_t + (1-s_t)e\right). \tag{2.1}$$

Notice that the type of good produced in the economy can be used as consumption good or as capital good invested in production. An agent's consumption is constrained by the after-tax working and/or capital income. In each period, workers and searchers receive different sources of income, so they have different budget constraints. A worker's budget constraint can be written as,

$$c_t + a_{t+1} \le (1-\tau)\left[y_t - r_t\left(k_t - a_t\right)\right] + (1-\delta)a_t + \mu_t, \tag{2.2}$$

here y_t is the total working income received. Since saving is undertaken in the form of physi capital, a capital owner has to pay capital income tax.

Similarly, the budget constraint for an unemployed agent is,

$$c_t + a_{t+1} \le (1-\delta)a_t + (1-\tau)r_t a_t + b_t + \mu_t. \tag{2.3}$$

As observed in the dynamic general equilibrium literature, the specific functional forms for U is chosen to be,

[10]If the model allows on-job search, an agent's expected lifetime utility can be written as,

$$\max_{\{c_t, a_{t+1}, k_t, l_t, e_t\}} E\sum_{t=0}^{\infty}\beta^t U\left(c_t, l_t + e_t\right),$$

where l_t is the time spent on working, and e_t is the time spent on job searching. Furthermore, if an unemployed worker searches more intensively, that is, a higher e_t, he has a better chance to find a good job opportunity.

$$U(c_t, l_t) = \frac{1}{1-\sigma}\left\{\left[c_t^{\theta}(1-l_t)^{1-\theta}\right]^{1-\sigma} - 1\right\}, \tag{2.4}$$

where σ and θ are positive real numbers, and the values used in the quantitative analysis will be discussed in the calibration section of the paper.

An agent's utility maximization problem is easily defined in the recursive fashion of dynamic programming. Obviously the employed and the unemployed have different value functions. The next subsection is devoted to the characterization of these value functions.

In addition, this model implicitly assumes that there is no cost associated with either the abandonment of an old production site or the adoption of a new site. Due to the computational difficulty, this model also assumes that there is no aggregate uncertainty.

2.2.2 Optimization Problems

The state of the world for an individual agent is completely summarized by his asset holdings, a, technology shock, z, the rental rate on capital, r, the lump-sum transfer, μ, and unemployment insurance benefit, b. In turn, (r, μ, b) are determined by market clearing conditions and are dependent on the equilibrium distribution of asset holdings and production opportunities across agents, F. So only the individual state variables (a, z) and the joint distribution matter for the characterization of an agent's optimization problem.

So for workers who stand in state (a, z), the Bellman equation will be

$$W(a,z) = \max_{\{a',c,k,l\}} \{U(c,l) + \beta \int \max[W(a',z'), S(a')] G(dz',z)\}$$

$$s.t. \begin{cases} c + a' \leq (1-\tau)zk^{\alpha}l^{1-\alpha} + (1-\delta)a + (1-\tau)r(a-k) + \mu, \\ \\ a, c, \text{and } k \geq 0, \text{ and } 1 \geq l \geq 0, \end{cases} \tag{2.5}$$

where primed variables denote the next period values and $S(.)$ is the corresponding value function for searching agents. The function F is suppressed in the value functions, W and S, for notational simplicity.

Since this is a standard programming problem, the budget constraint will hold with equality in equilibrium. Then problem (2.5) can be simplified to

$$\begin{aligned} W(a,z) &= \max_{\{a',k,l\}} \{U((1-\tau)zk^{\alpha}l^{1-\alpha} + (1-\delta)a + (1-\tau)r(a-k) \\ &\quad + \mu - a', l) + \beta \int \max[W(a',z'), S(a')] G(dz',z)\}. \end{aligned} \tag{2.6}$$

Given the preference specified above, the optimal k and l for a worker can be solved as a function of the equilibrium rental rate r^*, the tax rate, τ, the asset holdings, (a, a'), and the idiosyncratic shock, z. Denote the functions for the optimal capital and labor inputs employed by a worker by $k^* = k^*(z, a; r^*, \tau, a')$ and $l^* = l^*(z, a; r^*, \tau, a')$, then

$$l^* = \frac{\Delta_1}{\Delta_2}, \tag{2.7}$$

and

$$k^* = \left[\frac{\alpha z}{r^*}\right]^{\frac{1}{1-\alpha}} \frac{\Delta_1}{\Delta_2}, \tag{2.8}$$

where

$$\begin{aligned} \Delta_1 &= a' + \theta z (1-\tau) \left(\tfrac{1-\alpha}{1-\theta}\right) \left[\tfrac{\alpha z}{r^*}\right]^{\frac{\alpha}{1-\alpha}} \\ &\quad - (1-\delta) a - (1-\tau) r^* a - \mu, \end{aligned} \tag{2.9}$$

and

$$\Delta_2 = z (1-\tau) \left(\tfrac{1-\alpha}{1-\theta}\right) \left[\tfrac{\alpha z}{r^*}\right]^{\frac{\alpha}{1-\alpha}} \tag{2.10}$$

From the above two equations, it is easy to see that the amount of capital and labor employed are dependent on a worker's current period asset holdings, a', positively and last period asset holdings, a, negatively. The lump-sum transfer has a negative effect on a worker's labor supply and capital demand. However, the interest rate has an ambiguous effect on l^* and k^*. So does the income tax rate.

By simple calculation, the after-tax real income a worker earns from operating a site with productivity shock z, denoted $y_w(z, a, \mu, a')$, reads

$$\begin{aligned} y_w(z,a,\mu,a') &= \alpha \cdot z \left[\frac{\alpha z}{r^*}\right]^{\frac{\alpha}{1-\alpha}} \frac{\Delta_1}{\Delta_2} \\ &= \frac{\alpha(1-\theta)}{(1-\alpha)(1-\tau)}\Delta_1 \end{aligned} \tag{2.11}$$

So a worker's after-tax income increases with the idiosyncratic disturbance and decreases with the equilibrium interest rate directly. How does income tax affect work effort and job search? From equations (2.11) and (2.9), it is certain that the income taxes have negative effect on $y_w(.)$ when a' drops faster than $(1-\tau)$ does.[11] In addition, the capital income tax drives up the before tax interest rate and reduces the wage income further. These negative effects of income taxes on $y_w(.)$ make all jobs less attractive to job hunters. Thus income taxes discourage work and job search.

Now the Bellman equation for a worker in state (a, z) can be further simplified to

$$\begin{aligned} W(a,z) &= \max_{\{a'\}} \{U((1-\tau)z(k^*)^{\alpha}(l^*)^{1-\alpha} + (1-\delta)a + \\ &\quad (1-\tau)r(a-k^*) + \mu - a', l^*) \\ &\quad +\beta \textstyle\int \max\left[W(a',z'), S(a')\right] G(dz',z)\}. \end{aligned} \tag{2.12}$$

Analogously, the Bellman equation for unemployed agents in state (a, z) reads

[11] This is a sufficient condition only, and it implies the case where private savings is quite sensitive to the income tax rate.

$$S(a) = \max_{\{a',c\}} \left\{ U(c,e) + \beta \int \max\left[W(a',z'), S(a')\right] \Phi(dz') \right\}$$

$$s.t. \quad \begin{cases} c + a' \leq (1-\delta)a + (1-\tau)ra + b + \mu, \\ c, e \geq 0, \end{cases} \tag{2.13}$$

which can be simplified to

$$S(a) = \max_{\{a'\}} \left\{ U\left((1-\delta)a + (1-\tau)ra + b + \mu - a', e\right) \right.$$
$$\left. + \beta \int \max\left[W(a',z'),\ S(a')\right] \Phi(dz') \right\}. \tag{2.14}$$

From this Bellman equation, it is easy to see that an unemployed worker will be more likely to accept a new job opportunity when the rate of income tax is increased. Simultaneously, as shown before, income taxes reduce the income, $y_w(.)$, for all z. The average expected return to job search decreases. Generally speaking, proportional income taxes discourage job search and result in a lower search unemployment rate.

Given τ, μ and b, the value functions (W, S) are non-decreasing in last period assets. Also, W and S are the increasing functions of the government lump-sum transfer. An unemployed worker is better off with a higher unemployment compensation and lower tax rates. With Blackwell's sufficient conditions for a contraction mapping, equation (2.12) and equation (2.14) have a unique fixed point (W^*, S^*).[12]

2.2.3 Stationary Recursive Competitive Equilibrium

Given a set of government fiscal policy parameters, (ρ, τ), and a stationary distribution function $F^*(a, z)$, all aggregate variables will stay constant in a stationary equilibrium.

In a steady state, the equilibrium rental rate clears the capital market, i.e.,

$$\int_{A\times Z} \Psi\left\{l^* > 0\right\} \cdot k^* dF^* = \int_{A\times Z} a dF^*. \tag{2.15}$$

where $\Psi\left\{x\right\}$ is an indicator function, which takes the value 1, if the condition x is satisfied, 0 otherwise.

The left hand side of the above equation represents the aggregate capital demand, and the right hand side of the equation the total amount of capital supplied in the economy. Furthermore, the market clearing condition plus the individual budget constraints imply an aggregate resource constraint.

By definition of equilibrium, the government budget is balanced; total UI benefits and other transfers equal total tax revenue,

[12]This can be easily verified by theorem 9.7 in Stokey et al. (1989).

$$
\begin{aligned}
\textstyle\int_{A\times Z}[b^*(1-\Psi\{l^*>0\})+\mu^*]dF^* \quad = \quad & \textstyle\int_{A\times Z}\tau\left[z(k^*)^{\alpha}(l^*)^{1-\alpha}+r^*(a-k^*)\right]\cdot \\
& \Psi\{l^*>0\}\,dF^*+ \qquad\qquad (2.16) \\
& \textstyle\int_{A\times Z}\tau r^* a\left[1-\Psi\{l^*>0\}\right]dF^*,
\end{aligned}
$$

The left hand side of the above equation represents the total government expenditure; the right hand side of the equation equals the revenue from income taxation paid by both workers and searchers.

The equilibrium unemployment insurance benefit, b^*, can be calculated by the following formula,

$$
b^* = \rho\left\{\frac{\int_{A\times Z}\Psi\{l^*>0\}\,y_w(z)dF^*}{\int_{A\times Z}\Psi\{l^*>0\}\,dF^*}\right\}, \tag{2.17}
$$

where individual wage income, $y_w(z)$, is defined in equation (2.11). Accordingly, the numerator in the above formula is the total after-tax work income and the denominator is the number of working agents. Here, the average after-tax wage income from work is used to approximate the average after-tax income from work received by the unemployed agents before they lose their jobs.

Finally, the law of motion for the joint distribution of assets and productivity is described by

$$F^*(A_0 \times Z_0) = \int_{A\times Z}\left\{\int_{Z_0}\chi\left\{l^*>0,(a')^*\in A_0\right\}G(dz',z)\right\}dF^*$$

$$+ \int_{A\times Z}\left\{\int_{Z_0}\tilde{\chi}\left\{l^*=0,(a')^*\in A_0\right\}\Phi(dz')\right\}dF^*, \tag{2.18}$$

$$\text{for all } (A_0 \times Z_0) \subseteq A \times Z,$$

where χ and $\tilde{\chi}$ are two indicator functions.

The first term in the right hand side of the above equation represents the fraction of agents, with state, (a, z), who choose to produce and hold assets $(a')^* \in A_0$, given the transition probability of their productivity from z to $z' \in Z_0$. The second term is the fraction of agents who find a new production opportunity, $z' \in Z_0$, with probability $\Phi(z')$ and hold assets $(a')^* \in A_0$ optimally in the next period.

A *stationary recursive competitive equilibrium* is a price, $r^*(\tau, \rho)$, a sequence of allocation functions, $l^*(z; \tau, \rho)$, $k^*(z; \tau, \rho)$, and $(a')^*(a, z; \tau, \rho)$, a distribution function over assets and productivity across agents, $F^*(a, z; \tau, \rho)$, a lump-sum transfer, $\mu^*(\tau, \rho)$, and a uniform unemployment insurance benefit , $b^*(\tau, \rho)$, for a given set of government policy parameters, (τ, ρ), such that:

- Given the price and social transfer, $l^*(z; \tau, \rho)$ and $k^*(z; \tau, \rho)$ are given by equations (2.8) and (2.7). Furthermore, $(a')^*(a, z; \tau, \rho)$ solves agents' optimization problems (2.12) and (2.14).

- The capital rental rate, $r^*(\tau, \rho)$, satisfies equation (2.15).
- The equilibrium distribution, F^*, is the fixed point to equation (2.18).
- The government budget is balanced by $\mu^*(\tau, \rho)$ and $b^*(\tau, \rho)$.

Once the above stationary equilibrium has been computed, it is easy to calculate the value for any variable of interest. For instance, the equilibrium unemployment rate can be written as a function of workers' employment decisions and F^*,

$$U_{rate} = \int_{A \times Z} [1 - \Psi \{l^* > 0\}] \, dF^*. \tag{2.19}$$

2.3 Calibration

The model economy is calibrated so that the period length is equal to six weeks. Given the length of the period, the value of the discount factor, β, is set to be .995. Likewise, a value of .0125 is chosen for the capital depreciation rate, δ, under the assumption that the annual rate is .10. [See Prescott (1986).] Capital's share of the income parameter, α, is set to .36 [again, see Prescott (1986)].

As suggested by Gruber's result, the optimal UI benefit level is within the range of current replacement rates only at fairly high levels of risk aversion. The coefficient of relative risk aversion, σ, is set to be 2.0. The value of θ is chosen to be .855 which is close to that used in Krusell and Smith (1995). According to Barron and Mellow (1979), the mean number of hours spent searching per week is approximately seven, and Americans on average work 39 hours per week. So the leisure cost of searching is set such that the ratio of the search time to the average working time is equal to .18, which is compatible with the data.

In the model, there exist three levels of productivity. The distribution drawn by the unemployed is set in the following way. The escape rate from unemployment into employment equals 52%, such that the average duration of unemployment in the benchmark case with UI is about 12 weeks, which is consistent with that observed in the U.S. economy (see Meyer, 1990). The corresponding weekly exit rate, 8.7%, falls into the range [3.659%, 9.917%] which has been found by Meyer (1990) in the U.S. economy. Under the assumption that about 10% of the agents exiting from unemployment draw the highest productivity and the rest draw the medium level productivity, this leaves the approximated distribution drawn by an unemployed worker to be $\Phi(z') = [.48, .47, .05]$. Three productivity levels are chosen to be $Z = [0.55, 0.9, 1.0]$, where the medium productivity level is chosen such that capital-output ratio is 3.0 (see Huggett, 1994). The Markov transition matrix for a worker's productivity is constructed from NLSY data [1978-1989][13],

---

[13]The transition matrix, G, is constructed by the following procedure. At survey year t, each surveyed worker belongs to either the low wage group, $W_l(t) = [\$0, \$5.00)$, or the medium wage group, $W_m(t) = [\$5.00, \$20.00)$, or the high wage group, $W_h(t) = [\$20.00, \$99.99]$. Let $\widehat{g}_{ij}(t, t+1)$ be the fraction of agents moving from wage group $W_i(t)$ into wage group $W_j(t+1)$, that is, $W_j(t+1)/W_i(t)$, where $i, j \in \{l, m, h\}$. Then the average value of $\widehat{g}_{ij}(t, t+1)$ for all survey years, $\frac{1}{N-1}\sum_{t=1}^{N-1} \widehat{g}_{ij}(t, t+1)$, is used as the approximate of the transition probability from productivity level i to j, g_{ij}, where $G = (g_{ij})$.

$$G\left(z' \mid z\right) = \begin{pmatrix} .6795 & .2132 & .1073 \\ .0559 & .9000 & .0441 \\ .0500 & .1037 & .8463 \end{pmatrix}.$$

Since agents are not allowed to borrow from each other, this implies that the lower bound of assets is 0. The upper bound, 4, is chosen such that, in equilibrium, no agents would hold more than this amount of assets.

In the benchmark model, the income tax rate is set to be .30, which is compatible with the U.S. economy, [see Greenwood and Huffman (1991)]. This is approximate because both the labor and capital income tax rates are chosen to be equal. The benchmark replacement ratio is set at .45 which is close to the average value, .426, found by Gruber (1994) from PSID data.

Once the model is calibrated, the equilibrium allocations can be solved numerically by the algorithm described in Appendix II. In addition, the statistics of interested are calculated according to the set of measurements given in the same appendix.

2.4 Findings

2.4.1 Qualitative Analysis

On the one hand, UI benefits have positive effects on unemployed agents' welfare. On the other hand, UI itself and the taxes, which finance a UI program, also create distorting effects on agents' working, searching and saving decisions.

For workers, a positive lump-sum transfer induces them to work less and switch/quit jobs more frequently due to a smaller searching cost; for job searchers, an increase in UI benefits will allow them to search longer and consume more due to income effect. Therefore, an increase in UI benefits directly increases the average duration of unemployment.

In this model, income taxes affect *both* unemployed and employed workers by lowering their income. Equations (2.9) and (2.11) imply that, under certain condition(s), income taxes directly lower all levels of wage income, y_w, so that the return to job search becomes smaller. In addition, capital income tax discourages private saving. So high tax rates reduce both wage income and capital income. Therefore, workers are less likely to quit their jobs, and unemployed workers hold out less long for better jobs at high tax rates.

2.4.2 Quantitative Experiments

This subsection discusses the design and implementation of a set of experiments. The experiments evaluate the distorting effects and the benefits of various unemployment insurance programs. The distorting effects include the increases in average unemployment rate and unemployment duration; the benefits are the gains of consumption smoothing and job search for unemployed workers.

More specifically, the benchmark experiments will quantify these benefits and costs in two economies. In the first economy, there is no UI, but the government will collect income tax (at the rate of 0.3) from all agents and rebate the tax revenue back to consumers in a lump-sum fashion. In the second economy there is a UI program, which is characterized by the replacement rate, 0.45. In the second economy the income tax rate is endogenously determined such that the UI program can be financed by the extra tax revenue raised. Therefore agents in both economies receive the same amount of lump-sum transfer from the government.

Other experiments are similar to the benchmark exercise except they are designed to examine the changes when a UI program becomes more generous or when there are different assumptions about the income tax rate.

Scenario I: The Benchmark Case

The benchmark experiments are conducted to determine whether it is worthwhile to introduce an unemployment insurance program in a search economy. Quantitative results for these two experiments can be found in the first two columns of Table 2.1 – 2.3.

Without insurance, agents have to insure themselves against potential downturns. Once unemployed workers have the chance to receive UI from the government, their mean consumption increases by 14.2% from the benchmark level without UI. [See Table 2.1.] This indicates that unemployment insurance does fulfill its main task as insurance for workers who encounter unpredictable downturns in their productivity and are in transition from old jobs to new jobs. Does UI have positive effects on job search? The answer is yes. The average wage received by workers who exit unemployment increases by 14.0% from that in economy 1. [See Table 2.1.] So job searchers do experience gains from the provision of unemployment insurance. This is because UI allows the unemployed agents, on average, to search longer for better job opportunities. As shown in Table 2.2, the average unemployment duration is increased from 8.9 weeks to 11.5 weeks.

The increase in the unemployment duration is a conventional target for those who criticize conventional UI policy. In this search model, such a cost becomes the source of the gain in job search for those unemployed people. UI affects not only the unemployed's job search behavior, which is characterized by the change in average unemployment duration (or job finding rate), but also the potential job switcher's behavior, which is described by the change in the job quit rate[14]. By simple calculation, the job finding rates are 0.112 and 0.086 in the cases without UI and with UI respectively. So the drop in the job finding rate can be mapped into the 1.16 percentage points increase in the unemployment rate if the job quit rate remains unchanged. Then the remaining 4.33 percentage point increase in the unemployment

[14]The job finding rate for unemployed workers can be approximated by $1/ud$ where ud is the average unemployment duration. In the steady state, the job finding rate, f, and the job quit rate, q, satisfy the following formula,

$$f \cdot u = q \cdot (1 - u) \text{ or } u = q/(f + q).$$

So the total change in the unemployment rate can be decomposed into the change due to the variation of the job finding rate for the fixed job quit rate and the change due to the variation of the job quit rate.

I thank Andreas Hornstein for pointing out this to me.

Table 2.1: The Effects of U.I. on Consumption and Job Search

Economies	[1]	[2]	[3]	[4]	[5]	[6]	[7]
type	No UI	With UI	With UI	No UI	With UI	No UI	With UI
income tax rate	0 3	0 268	0 273	0 25	0 259	0 35	0 276
UI replacement rate	n.a.	0.45	0.50	n.a.	0.45	n.a	0.45
U.I. Benefits	n a.	0 0907	0.1064	n a	0 0899	n.a.	0.0908
Lump-Sum Transfer	0 2178	0 2178	0.2178	0 2141	0 2141	0 2245	0.2245
Searcher's Consumption (Mean)	0 6352	0.7259 [14 2%]	0.7881 [24 0%]	0.7603	0.8282 [8.9%]	0 5607	0 7094 [26 5%]
Average Wage (of those exiting unemployment)	0 6700	0.7643 [14 0%]	0.7659 [14 3%]	0 7231	0.7653 [5.8%]	0.6332	0 7582 [19 7%]

NOTE: The numbers in the square brackets are the percentage changes from the corresponding levels in the case without UI.

Table 2.2: The Distorting Effects of U.I. on Unemployment

Economies	[1]	[2]	[3]	[4]	[5]	[6]	[7]
type	No UI	With UI	With UI	No UI	With UI	No UI	With UI
income tax rate	0 3	0.268	0 273	0.25	0 259	0.35	0.276
UI replacement rate	n a.	0.45	0.50	n.a.	0.45	n.a.	0.45
Unemployment Rate (%)	4 00	9.49	9 44	5.19	9.55	2.50	9 38
Avg. Unemploy't Duration [weeks]	8 9	11 5	11 6	9 9	11.5	8 1	11 4

NOTE The numbers in the square brackets are the percentage changes from the corresponding levels in the case without UI.

Table 2.3: Social Welfare and Redistributive Effect

Economies	[1]	[2]	[3]	[4]	[5]	[6]	[7]
type	No UI	With UI	With UI	No UI	With UI	No UI	With UI
income tax rate	0 3	0 268	0 273	0 25	0 259	0 35	0 276
UI replacement rate	n.a	0.45	0.50	n.a	0.45	n.a.	0.65
Social Welfare	-0 2819	-0 1462	-0 0972	0 1843	-0 0776	-0 3335	-0 1585
in terms of % change in avg consumption		[1 1%]	[1 5%]		[0 9%]		[1 4%]
Gini Coefficient	0 26	0 23	0 22	0 28	0 18	0 21	0 23
% of agents with 0 wealth	2 1	0 2	0 1	0 9	0 1	4 3	0 3

NOTE: The numbers in the square brackets are the percentage changes from the corresponding levels in tha case without UI rate is due to the increase in the job quit rate. So the increase in the unemployment rate is mostly attributed to new entrants.

The introduction of UI results in fewer agents being liquidity constrained. The reasons are the following: First, UI eases poor unemployed agents' financial burden. Second, the fraction of agents who have no assets drops from 2.1% to 0.2%, as shown in Table 2.3. Observe that the equilibrium income tax rate has fallen from 0.3 to 0.268. This indicates that a lower equilibrium income tax rate leaves fewer people at the lower end of the wealth distribution, since the net return on savings is higher. Consequently, the Gini coefficient decreases from 0.26 to 0.23. Therefore, the UI program results in lower income inequality.

The introduction of UI brings both costs and benefits to the economy. The increase in social welfare, from −0.2819 to −0.1462, indicates that the benefits dominate the costs of the program with a fixed lump-sum transfer. [See Appendix II for the formal definition of social welfare.] This translates into 1.1% increase in average consumption.

Scenario II: Variations in the Replacement Ratio

The purpose of the next experiment is to examine how an unemployment insurance program is affected as the degree of its generosity changes. In this experiment, the replacement ratio is set at 0.5 as opposed to the benchmark level of 0.45. Again, income taxes are changed in order to finance the program. The experiment results are listed in the third columns of Table 2.1–2.3.

As shown in the first three columns of Table 2.1, when the UI program becomes more generous, the positive effect of UI on the unemployed's consumption becomes more dramatic, and the effect on job search is also bigger. Specifically, the unemployed workers, on average, consume 9.8% more under a more generous program, which is consistent with the empirical findings by Gruber (1994).

On the cost side, the average unemployment duration is increased by 0.1 weeks due to a 5 percentage point increase in the replacement rate. However, a more generous UI program is associated with a higher income tax rate. Along with the new higher income tax rate of 0.273 (as opposed to 0.268), the unemployment rate falls from 9.49% to 9.44%. Fewer workers enter the unemployment pool at a higher equilibrium tax rate. This is shown by the fall in the job quit rate from 0.9% to 0.7%. For those workers on the margin, working is a better choice than searching for new jobs in the current situation. These results demonstrate that: (1) A more generous program does not necessarily give rise to a higher unemployment rate; (2) When the UI compensation is not significant in the workers' consumption, the distorting effect of UI will be limited.

A more generous program increases the unemployed workers' average consumption. Consistently, the Gini coefficient becomes smaller, dropping from 0.23 to 0.22. Finally, the statistics in Table 2.3 shows that, within a reasonable range, a more generous program is preferable in terms of social welfare.[15] This is because the additional cost of this more generous program is relatively small compared with the additional benefits it generates.

Scenario III: Variations in Income Tax Rate

The following set of experiments considers introducing UI into three economies, 1, 4, and 6, characterized by three different income tax rates, 0.30, 0.25, and 0.35. The UI program in each of the three economies, 2, 5, and 7, is characterized by the benchmark replacement rate of 0.45. These experiments are designed to explore how the economic system responds to the introduction of UI under various income tax rates. The results for these experiments are given in the corresponding columns of Table 2.1–2.3.

As shown in Table 2.1, introducing UI increases the gains to the unemployed's consumption and job search monotonically as the tax rate increases.[16] This is because higher tax rates make the unemployed workers poorer and less willing to be patient and search for better job opportunities. Therefore the unemployed agents would benefit more from the provision of UI.

How does the introduction of UI, under various tax rates, affect job search behavior? Let $\Delta u(\tau)$ denote the change of unemployment rate, where $\Delta u(\tau) = u(\tau^*) - u(\tau)$,

[15]Since optimal unemployment insurance is not the main focus of this paper, searching for this reasonable range is not the task here.

[16]Gains to the unemployed's job search are characterized by the increase in average wage received by those exiting unemployment.

and $u(\tau)$ and $u(\tau^*)$ are the unemployment rates before and after introducing UI in an economy. It is easy to see from Table 2.2 that $\Delta u(\tau)$ increases in the income tax rate, τ.[17] In detail, the variation of the unemployment rate can be decomposed into the corresponding increase in the job quit rate, $\Delta q(\tau)$, and the decrease in the job finding rate, $\Delta f(\tau)$. The statistics in the following table characterize the changes in the job finding rate and the job quit rate due to the changes of income tax rates. The results indicate that introducing UI at a higher income tax rate induces more workers to quit their jobs and, on average, allows job searchers to stay longer in the unemployment pool.

τ (before introducing UI)	0.25	0.3	0.35
$\Delta u(\tau)$ [%]	4.36	5.49	6.88
$\Delta q(\tau)$ [%]	0.37	0.44	0.59
$\Delta f(\tau)$ [%]	-1.4	-2.5	-3.6

Finally, the changes in social welfare, as reported in Table 2.3, indicate that an economy benefits more from the introducing of UI at high tax rates.[18]

[17] In the regimes without UI, higher tax rates drive more agents out of the searching pool. This can be seen by the decrease in the unemployment rate from 5.9% to 4.0% and then to 2.5% in Table 2.2. This phenomenon is also observed in the cases with UI.

[18] If there are no income taxes in the economy, it is expected that the capital stock would be inefficiently high due to precautionary savings. However, when income taxation is introduced in an economy, as shown by the experiment results, private saving is very sensitive to the income tax rate. Under such circumstances, this inefficiency will be limited.

2.5 Future Research

In order to have a thorough examination of the effects of an unemployment insurance program on job search, it is necessary to endogenize search intensity in this model. Specifically, it is assumed that agents know the conditional distributions of z_{t+1}, $\Phi(z_{t+1} \mid e_t)$, before searching, where the distribution functions are monotonically increasing in effort, e_t. So, the effort that a searcher would supply entirely determines the distribution which he might draw. Higher effort results in a better distribution in the following sense.

$$\int_Z g(z_{t+1}) d\Phi(z_{t+1} \mid e_t) > \int_Z g(z_{t+1}) d\Phi(z_{t+1} \mid e'_t), \tag{2.20}$$

for all $g(\cdot) \in C$, $\Phi(\cdot \mid \cdot) \in \Theta$, and $e_t > e'_t$, where C is the space of all continuous functions and Θ is the space of the distribution functions with certain properties. For instance, $\Phi(\cdot \mid e_t)$ is log-normal with mean $\overline{z_t}(e_t)$ and variance $\sigma^2_{z_t}(e_t)$, where the mean is defined to be the linear function of a searcher's effort, i.e.,

$$\overline{z_t}\,(e_t) = a + be_t.$$

Alternatively, a more sophisticated rule can be incorporated into the model, which allows agents to search for new production opportunities sequentially in a period. Each single search takes each agent a fixed amount of effort, e_t. Observe that, on average, the unemployed worker's reservation wage responds negatively to the length of the unemployment spell. To be consistent with this observation, it is necessary to assume that searching decays with the number of consecutive periods which an agent spends in the unemployment pool.

A more advanced search model needs to be developed in order to do more realistic policy implication analysis. In the more realistic case, workers search for better jobs, and firms provide vacancies to society, while working/searching and hiring decisions are made to maximize workers' utility and firms' profits, respectively. It is worthwhile to point out that this search model does not need a matching function as seen in the traditional search models.

Blanchard and Diamond (1989) showed that the in-flows to employment from both the non-labor force and the unemployment pool have almost the same size. This implies that a rational agent prefers other non-market activities to job search as the expected return to job search is too low. This issue can be discussed in a search model, which takes into account other non-market activities, such as home production.

In addition, the current model can be extended along other dimensions, by assuming a finite duration of UI benefits, and the integration of firm behavior. When the research focus is the business cycle fluctuation analysis, aggregate uncertainty has to be considered in the current search model.

APPENDIX II

An Iterative Algorithm

Given the recursive nature of the problem described in main text, an algorithm to solve for the stationary competitive equilibrium is designed in the following way. Observe that the joint distribution, F^*, does not affect directly an individual agent's optimal decisions on k, l, and a'. This simplifies the structure of this algorithm.

Specifically, make guesses on equilibrium interest rate, r_0, and lump-sum transfer, μ_0; substitute them into the optimization problems faced by both types of agents, that is, equations (2.12) and (2.14); make an initial guess for each of the value functions, W^0 and S^0; solve for optimal $x_0 \equiv ((a')_0,\ k_0, l_0)$ as the functions of r_0 and μ_0, for each point in state space, $A \times Z$. Then use r_0, τ_0 and x_0 to find the fixed point, F_0, to equation (2.18) . In addition, combine F_0 and x_0 to get a new interest rate, r_1, from equation (2.15) and a new lump-sum transfer μ_1, from the government budget constraint (2.16). Finally, go back to the individual agent's optimization problems and repeat the procedure until $(x_n,\ r_n\ , \mu_n, F_n)$ converges to a unique stationary equilibrium (x^*, r^*, μ^*, F^*).

Technically, the algorithm consists of the following steps.

1. A grid is defined over the model's state space, $A \times Z$. Specifically, $a \in [0,4]$ and

$z \in \{.55, .9, 1.0\}$. A grid for assets of 100 equally spaced points over interval, A, is chosen.

2. Initial guesses on r, μ, b, $W(.)$ and $S(.)$ are made.

3. For the given guesses, a value function iteration routine is employed to solve agents' optimization problems, (2.12) and (2.14) at each point of the discrete state space.

4. For the given interest rate and lump-sum transfer, use equation (2.18), combined with the solutions obtained from last step, to compute the new distribution over (a, z) s.

5. The interest rate is updated by the capital market clearing condition (2.15).

6. The unemployment insurance benefit and the lump-sum transfer are revised by formula (2.17) and government's budget constraint (2.16).

7. Check the convergence condition for each of the control variables, (a', k, l), the interest rate, r, the governmental lump-sum transfer, μ, and the distribution function, $F(a, z)$. The above procedure is repeated until all above variables and the distribution function have converged.

Measurement

All relevant statistics are computed based on the measurement defined below. In this model, the distorting effect of unemployment insurance on job search is represented by the average levels of unemployment and unemployment duration. The calculation of unemployment duration is based on the stationary transition probability from unemployment to unemployment, $P_{u,u}$,

$$U.D. = \frac{1}{1 - P_{u,u}}. \tag{2.21}$$

For an unemployed agent who has a units of assets in the first period of unemployment, the probability that he has been unemployed for n periods is

$$P^{n}_{u,u}(a) = P^{n-1}_{u,u}(a) \int_{Z} \Lambda \left\{ S\left(a^{n-1}(a)\right) > W\left(a^{n-1}(a), z'\right) \right\} \Phi\left(dz'\right), \tag{2.22}$$

where $a^{n-1}(a) = a^{*\prime}(a^{n-2}(a))$, $n = 2, \ldots, T$, $P^{1}_{u,u}(a) = 1$, Λ is the indicator function and $a^{*\prime}$ is this agent's equilibrium saving function.

Given the above probabilities, λ_n, the fraction of agents who have been unemployed for n periods, is computed according to the formula bel

$$\lambda_n = \int_{A \times Z} P^{n}_{u,u}(a) \left[1 - \Psi(l^* > 0))\right] F(da, dz). \tag{2.23}$$

Next, the paper presents the magnitude of the consumption smoothing effect generated by a UI program. This is measured by the variance of consumption with

UI

$$var(c_u) = \left\{\int_{A\times Z} (1-\Psi(l^*>0))\left[c_u^*(a)-\overline{c_u}\right]^2 F(da,dz)\right\} / \left\{\int_{A\times Z} (1-\Psi(l^*>0))\, F(da,dz)\right\}, \tag{2.24}$$

where

$$\overline{c_u} = = \left\{\int_{A\times Z} (1-\Psi(l^*>0))\, c_u^*(a) F(da,dz)\right\} / \left\{\int_{A\times Z} (1-\Psi(l^*>0))\, F(da,dz)\right\},$$

and

$$c_u^*(a) = (1-\delta)a + (1-\tau)r^*a + b^* + \mu^* - (a')^*(a).$$

The total savings by the workers receiving productivity shock, z, is

$$S(z) = \left\{\int_A \Psi\,(l^*>0)\,(a')^*(a,z)F(da,z)\right\}, \tag{2.25}$$

so the total savings by all workers is

$$S^w = \int_Z S(z)dz. \tag{2.26}$$

Similarly, the savin·s by all searchers is

$$S^s = \int_{A\times Z} (1-\Psi(l^*>0))\,(a')^*(a,z)F(da,dz). \tag{2.27}$$

Then the aggregate savings is

$$S = S^{w} + S^{s}. \tag{2.28}$$

Fourth, the average wage received by workers who just exit unemployment pool is,

$$\overline{\varpi} = \left\{ \int_{A\times Z} (1 - \Psi(l^{*} > 0))\, Q_{u,e}(a) \cdot F(da, dz) \right\} / N_{u,e}, \tag{2.29}$$

where $Q_{u,e}(a)$ is the expected wage received by a worker who would exit unemployment next period,

$$Q_{u,e}(a) = \int_{Z} \varpi(z')\Lambda \left\{ W\left(a'(a), z'\right) \geq S\left(a'(a)\right) \right\} \Phi\left(dz'\right), \tag{2.30}$$

$\varpi(z)$ is the wage received by a worker with productivity, z,

$$\varpi(z) = (1 - \alpha) z^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r^{*}}\right)^{\frac{\alpha}{1-\alpha}}, \tag{2.31}$$

and $N_{u,e}$ is the number of agents who would find jobs and exit the unemployment pool next period,

$$\begin{aligned} N_{u,e} \;&=\; \textstyle\int_{A\times Z} \left\{ \int_{Z} \Lambda \left[W\left(a'(a), z'\right) \geq S\left(a'(a)\right) \right] \Phi\left(dz'\right) \right\} \\ &\quad (1 - \Psi(l^{*} > 0))\, F(da, dz). \end{aligned}$$

Fifth, in this model the distributive effect of an UI program is summarized by the Gini coefficient. Before the calculation of the coefficient, it is necessary to know a worker's after-tax income,

$$I^w(a,z) = (1-\tau)z(k^*)^{\alpha}(l^*)^{1-\alpha} + (1-\delta)a + (1-\tau)r^*(a-k^*) + \mu^*, \qquad (2.32)$$

a searcher's after-tax income,

$$I^s(a,z) = (1-\delta)a + (1-\tau)r^*a + b^* + \mu^*, \qquad (2.33)$$

and the aggregate income,

$$I = \int_{A\times Z} \left[\Psi(l^*>0)I^w(a,z) + (1-\Psi(l^*>0))\, I^s(a,z)\right] F(da,dz). \qquad (2.34)$$

Furthermore, let the sorted income be $I' \in \left[\underline{I}, \overline{I}\right]$, then the share of aggregate income received by the bottom quantile, $\int_{\underline{I}}^{I_0} F(di)$, is

$$\text{Share}(I_0) \;=\; \int_{\underline{I}}^{I_0} I' F(di). \qquad (2.35)$$

Finally, the Gini coefficient can be computed by the standard formula

$$\text{Gini}(I_0) = \left\{\int_{\underline{I}}^{I_0} F(di) - \int_{\underline{I}}^{I_0} I' F(di)\right\} / \left[\int_{\underline{I}}^{I_0} F(di)\right]. \qquad (2.36)$$

Social welfare is given by

$$W = \int_{A\times Z} [\Psi(l^* > 0)\, U(c_e^*(a,z), l^*(a,z)) + \tag{2.37}$$

$$(1 - \Psi(l^* > 0))\, U(c_u^*(a,z), e)]\, F^*(da, dz).$$

The welfare change from the steady state level, W_b, is measured by the average percentage change of all agents' consumption, i.e.,

$$\Delta_w \equiv \frac{\Delta c}{E[c^*]},$$

such that,

$$W_b = \int_{A\times Z} [\Psi(l^* > 0)\, U(c_e^*(a,z) + \Delta_w E[c^*], l^*(a,z)) +$$

$$(1 - \Psi(l^* > 0))\, U(c_u^*(a,z) + \Delta_w E[c^*], e)]\, F^*(da, dz),$$

and

$$E[c^*] = \int_{A\times Z} [\Psi(l^* > 0)\, c_e^*(a,z) + (1 - \Psi(l^* > 0))\, c_u^*(a,z)]\, F^*(da, dz).$$

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